Master's Thesis Presentation

On the Parameterized Complexity of Semitotal Dominating Set on Graph Classes

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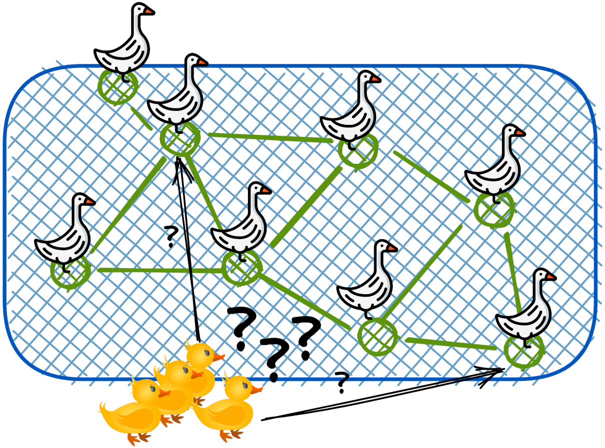
Theoretical Foundations of Artificial Intelligence
School of Computation
Technical University of Munich

February 28th, 2023

Quack!



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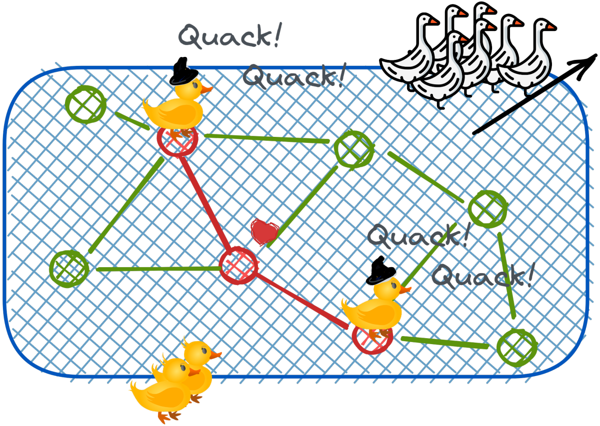
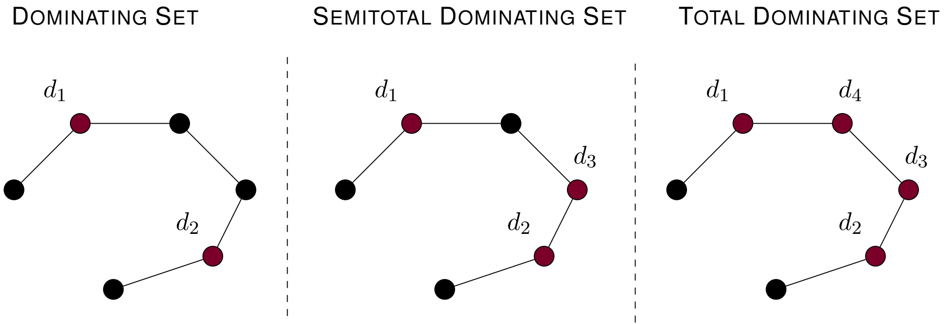


Our Plan for Today

Example: $\gamma(G) < \gamma_{t2}(G) < \gamma_t(G)$



Motivation

SEMITOTAL DOMINATING SET

Input	Graph $G = (V, E)$, $k \in \mathbb{N}$
Question	Exists ds $D \subseteq V$ with $\|D\| \leq k$ such that $\forall d_1 \in D : \exists d_2 \in D \setminus \{d_1\}$ with $d(d_1, d_2) \leq 2$?

- The semitotal domination number is the minimum cardinality of an sds of G, denoted as $\gamma_{t2}(G)$.
- **Observation:** $\gamma(G) \leq \gamma_{\mathbf{t2}}(\mathbf{G}) \leq \gamma t(G)$
- We say d_1 witnesses d_2 (and vice versa)

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Parameterized Complexity

- NP-hard? We expect problem to be **at least** exponential
- **Idea:** Limit combinatorial explosion to some aspect of the problem
- **Goal:** Find an algorithm running in time $\mathcal{O}(f(k) \cdot n^c)$ for **some** parameter k
- In this work: by solution size
- **Techniques:** Kernelization, Bounded Search Trees, ...

If possible, the problem is **fixed-parameter tractable**.

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Fixed-Parameter Intractability

- Class **NP** splits into whole hierarchy $W[i]$ in parameterized setting
- Problems at least $W[1]$-hard probably **fixed-parameter intractable**
- DOMINATING SET is $W[2]$-complete
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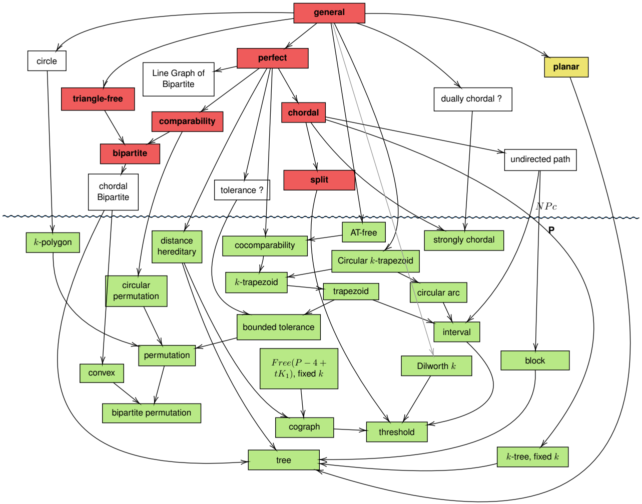
Complexity Landscape I

Graph Class	Dominating Set		Semitotal Dominating Set		Total Dominating Set	
	classical	Parameterized	classical	Parameterized	classical	Parameterized
bipartite	NPc [4]	$W[2]$ [40]	NPc [26]	$W[2]$ (We)	NPc [33]	?
line graph of bipartite	NPc [29]	?	NPc [19]	?	NPc [36]	?
circle	NPc [27]	$W[1]$ [7]	NPc [28]	?	NPc [36]	$W[1]$ [7]
chordal	NPc [6]	$W[2]$ [40]	NPc [26]	$W[2]$ (We)	NPc [38]	$W[1]$ [11]
s-chordal , $s > 3$	NPc [34]	$W[2]$ [34]	?	?	NPc [34]	$W[1]$ [34]
split	NPc [4]	$W[2]$ [40]	NPc [26]	$W[2]$ (We)	NPc [38]	$W[1]$ [11]
3-claw-free	NPc [14]	FPT [14]	?	?	NPc [36]	?
t-claw-free, $t > 3$	NPc [14]	$W[2]$ [14]	?	?	NPc [36]	?
chordal bipartite	NPc [37]	?	NPc [26]	?	P [15]	
planar	NPc [20]	FPT [2]	NPc	**FPT** (We)	NPc	FPT [21]
undirected path	NPc [6]	FPT [18]	NPc [25]	?	NPc [32]	?
dually chordal	P [8]		?[1]		P [31]	
strongly chordal	P [17]		P [41]		NPc [17]	
AT-free	P [30]		P [28]		P [30]	
tolerance	P [23]		?		?	
block	P [17]		P [25]		P [10]	
interval	P [12]		P [39]		P [5]	
bounded clique-width	P [13]		P [13]		P [13]	
bounded mim-width	P [3, 9]		P [19]		P [3, 9]	

Complexity Landscape II



Warmup: Intractability Results

$W[2]$-hard on split, chordal and bipartite graphs

- **Split Graph:** $G = \texttt{Clique} + \texttt{IndependentSet}$

Split Graphs

SEMITOTAL DOMINATING SET on *split* and *chordal* graphs is $W[2]$-hard

Proof by fpt-reduction from DOMINATING SET on split graphs:

1. **Observe**: Any ds D directly admits a sds D'.
2. Length of longest shortest path exactly 3
3. If $d \in (I \cap D)$, flip into K
4. Parameter $k' = k$

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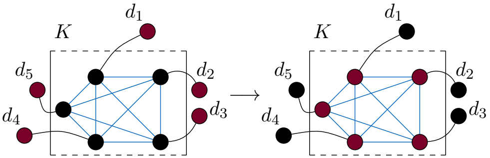

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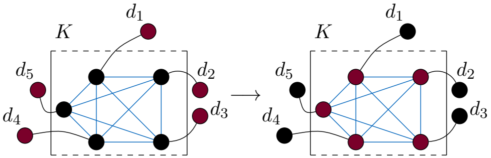

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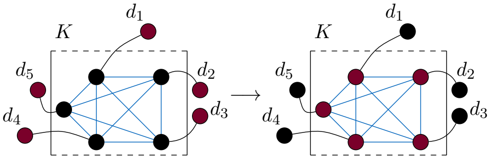

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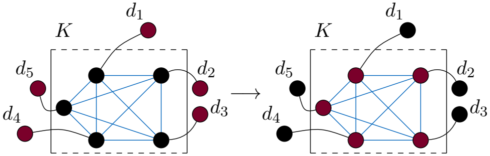

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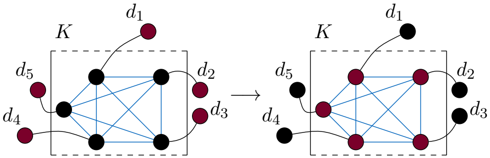

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Bipartite Graphs

SEMITOTAL DOMINATING SET on *bipartite* graphs is $W[2]$-hard

Proof by fpt-reduction from DOMINATING SET on bipart. graphs:

1. **Construct** Add new neighbor to each vertex and add d_1, d_2, u_1, u_2
2. If ds D in G, then $D' = D \cup \{d_1, d_2\}$ is sds in G'
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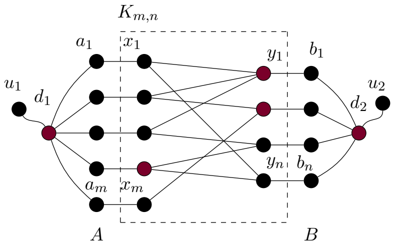

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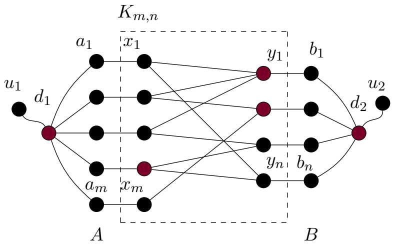

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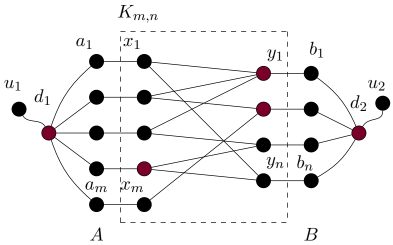

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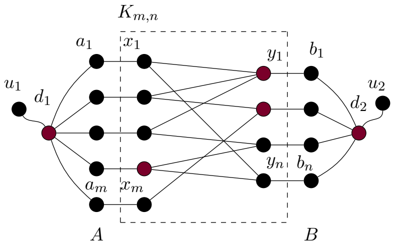

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A Linear Kernel for PLANAR SEMITOTAL DOMINATING SET

Kernelization

- **Idea:** Preprocess an instance using *Reduction Rules* until hard *kernel* bounded by $f(k)$ is found.

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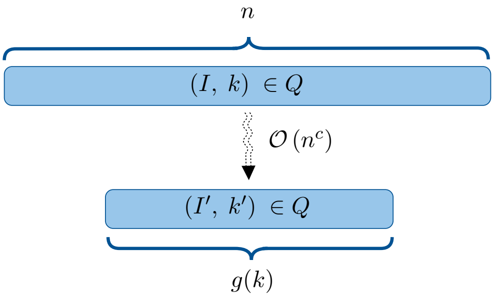

Related Works

Problem	Size	Source
PLANAR DOMINATING SET	$67k$	[16]
PLANAR TOTAL DOMINATING SET	$410k$	[21]
PLANAR SEMITOTAL DOMINATING SET	$358k$	Slide 18
PLANAR EDGE DOMINATING SET	$14k$	[24]
PLANAR EFFICIENT DOMINATING SET	$84k$	[24]
PLANAR RED-BLUE DOMINATING SET	$43k$	[22]
PLANAR CONNECTED DOMINATING SET	$130k$	[35]
PLANAR DIRECTED DOMINATING SET	Linear	[1]

Main Theorem

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PLANAR SEMITOTAL DOMINATING SET parameterized by solution size admits a linear kernel of size $|V(G')| \leq 358 \cdot k$.

The Big Picture

1. Split the neighborhoods of the graph $G = (V, E)$;
2. Define three reduction rules
3. Use a region decomposition to analyze the size of each region

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The Basic Principle: Regions

Region (Simplified)

Given plane G and $v, w \in V$, a region is a closed subset, such that

- there are two non-crossing (but possibly overlapping) boundary paths
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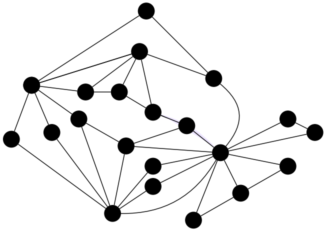

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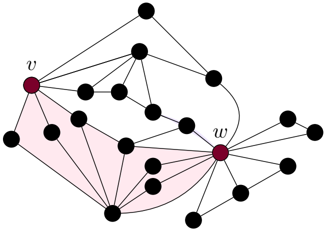
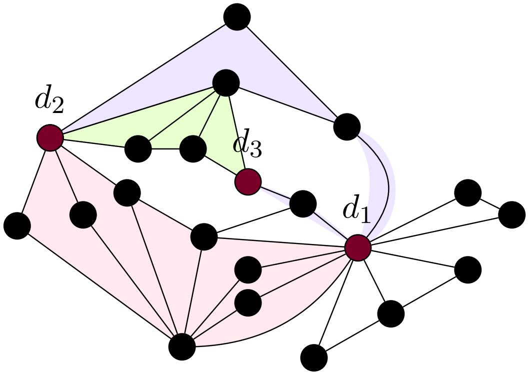


D-Region Decomposition



D-Region Decomposition (cont.)

D-*region decomposition* (Alber, Fellows, Niedermeier [2])

Given $G = (V, E)$ and sds $D \subseteq V$, a D-*region decomposition* is a set $\mathfrak{R}$ of regions with poles in D such that:

- The poles $v, w \in D \cap V(R)$ are only dominating vertices in the region.
- Regions are disjoint but can share border vertices

A region is **maximal**, if no $R \in \mathfrak{R}$ such that $\mathfrak{R}' = \mathfrak{R} \cup \{R\}$ is a D-*region decomposition* with $V(\mathfrak{R}) \subsetneq V(\mathfrak{R}')$.

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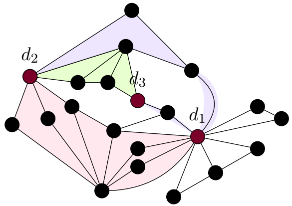
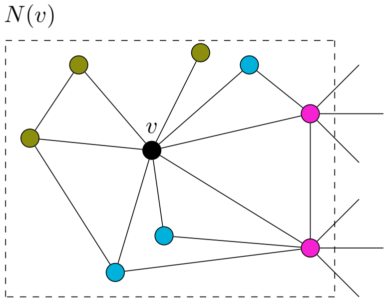


Splitting Up $N(v)$



We split $N(v)$ into three subsets:

$$N_1(v) = \{u \in N(v) : N(u) \setminus N[v] \neq \emptyset\} \quad (1)$$

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$$N_3(v) = N(v) \setminus (N_1(v) \cup N_2(v)) \quad (3)$$

For $i, j \in [1, 3]$, we denote $N_{i,j}(v) := N_i(v) \cup N_j(v)$.

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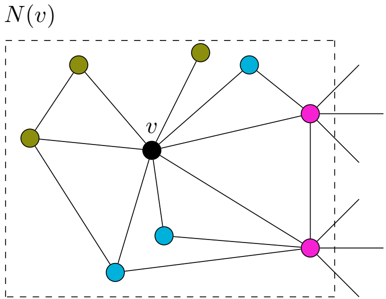

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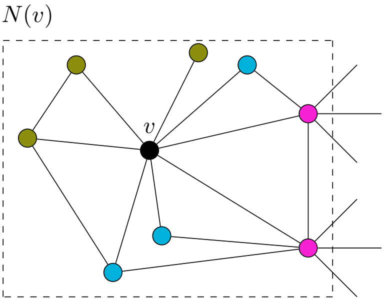

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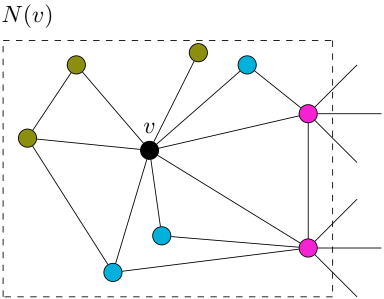

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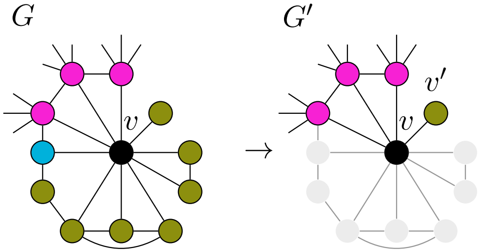
For $i, j \in [1, 3]$, we denote $N_{i,j}(v) := N_i(v) \cup N_j(v)$.

Rule 1: Shrinking $N_3(v)$

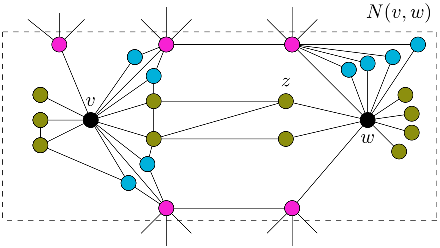
Let $G = (V, E)$ be a graph and let $v \in V$. If $|N_3(v)| \geq 1$:

- remove $N_{2,3}(v)$ from G,
- add $\{v, v'\}$.



- **Idea:** v better choice than $N_{2,3}(v)$

Splitting up $N(v,w)$



$$N_1(v,w) = \{u \in N(v,w) \mid N(u) \setminus (N(v,w) \cup \{v,w\}) \neq \emptyset\} \tag{4}$$

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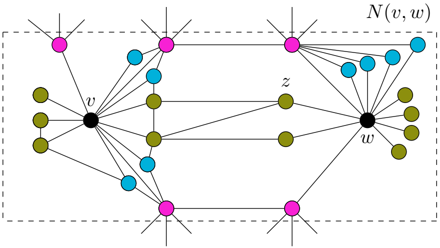

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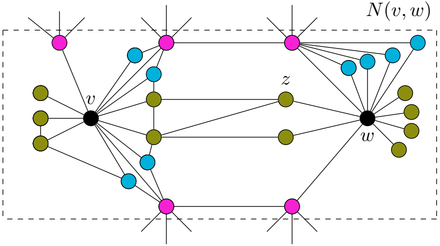

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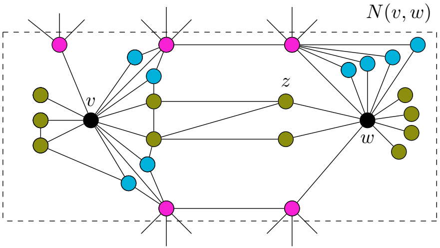

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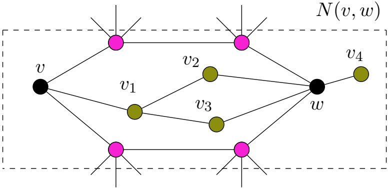

$$\mathcal{D} = \{\tilde{D} \subseteq N_{2,3}(v, w) \mid N_3(v, w) \subseteq \cup_{v \in \tilde{D}} N(v),\ |\tilde{D}| \leq 3\} \tag{7}$$

$$\mathcal{D}_v = \{\tilde{D} \subseteq N_{2,3}(v, w) \cup \{v\} \mid N_3(v, w) \subseteq \cup_{v \in \tilde{D}} N(v),\ |\tilde{D}| \leq 3,\ v \in \tilde{D}\} \tag{8}$$

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Key Idea: $N_{2,3}(v, w)$ can **always** be semitotally dominated with 4 vertices.
Lemma: $\mathcal{D} = \emptyset$ and $\mathcal{D}_v = \emptyset$, then any solution contains w. Simply neighborhood.

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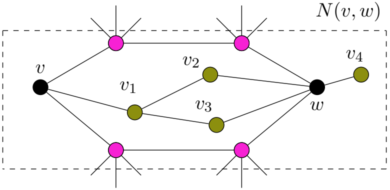

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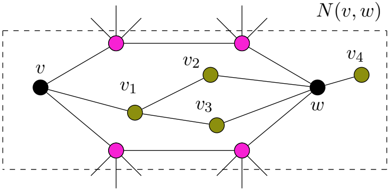

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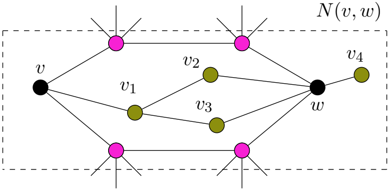

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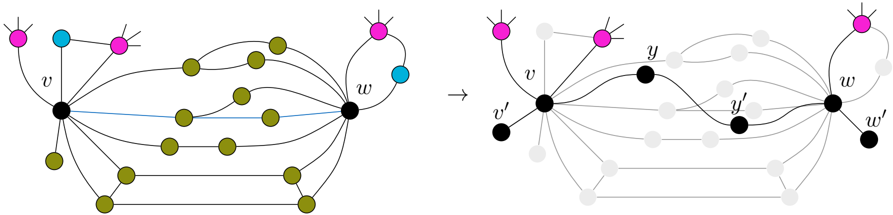
Rule 2

Case 1: If $\mathcal{D} = \emptyset$ and $\mathcal{D}_v = \emptyset$ and $D_w = \emptyset$

- Remove $N_{2,3}(v, w)$
- Add vertices v' and w' and two edges $\{v, v'\}$ and $\{w, w'\}$
- Preserve $d(v, w)$

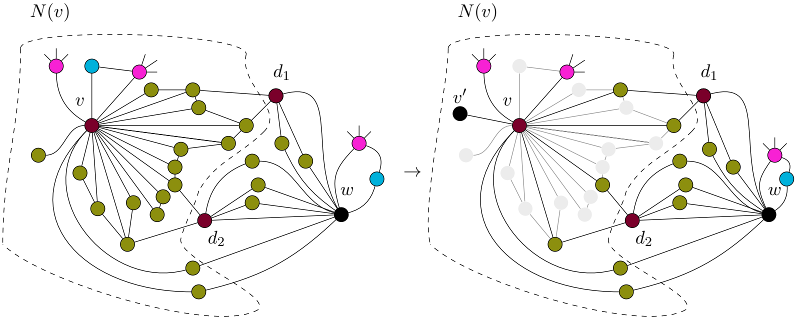


Rule 2

If $\mathcal{D} = \emptyset$ we apply the following:

Case 2/3: if $\mathcal{D} = \emptyset$ and $\mathcal{D}_v \neq \emptyset$ and $D_w = \emptyset$

- Remove $N_{2,3}(v)$
- Add $\{v, v'\}$



Simple Regions

Simple Region [21]

A simple vw-region is a vw-region such that:

1. its boundary paths have length at most 2, and
2. $V(R) \setminus \{v, w\} \subseteq N(v) \cap N(w)$.

Rule 3: Shrinking simple region to at most 4 vertices + preserving witness properties.

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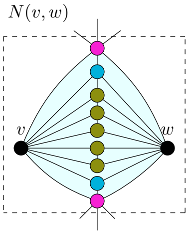

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We proved, that

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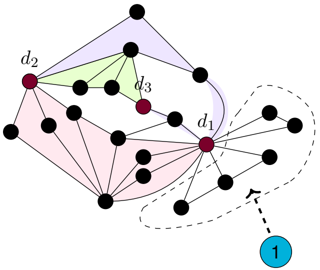

For each d in sds D:

1. $|N_1(v) \setminus V(\mathfrak{R})| \leq 0$ [2], On Border
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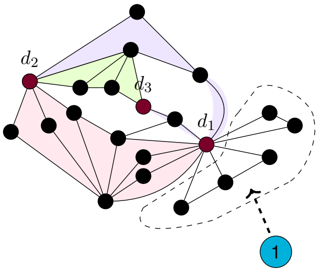

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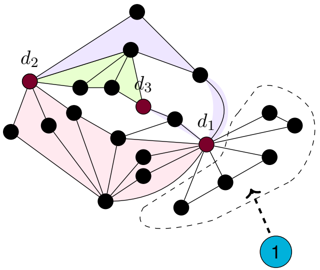

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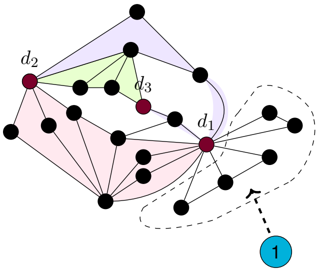

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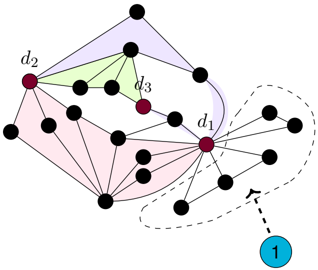

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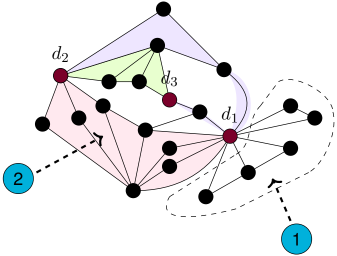

For each vw-region, we have

1. $|N_1(v,w)| \leq 4$ (vertices on border [2])
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Total: $|V(R)| = |\{v,w\} \cup (N_1(v,w) \cup N_2(v,w) \cup N_3(v,w))| \leq 87$

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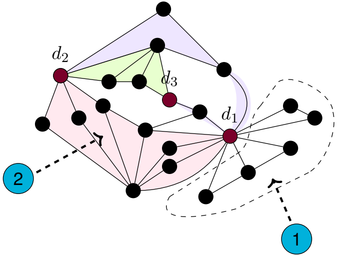

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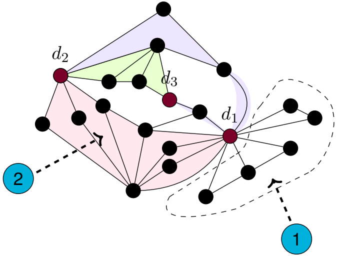

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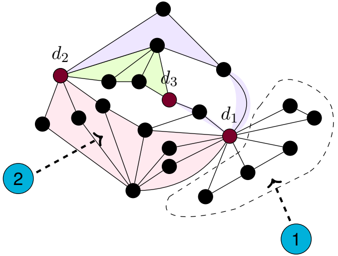

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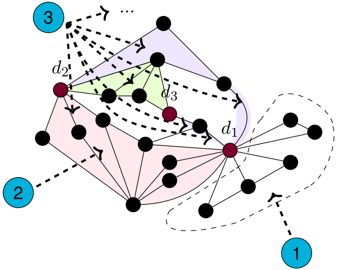

Number of Regions [2]

Let G be a plane graph and let D be a SEMITOTAL DOMINATING SET with $|D| \geq 3$. There is a maximal *D-region decomposition* of G sucht that $|\mathfrak{R}| \leq 3 \cdot |D| - 6$.

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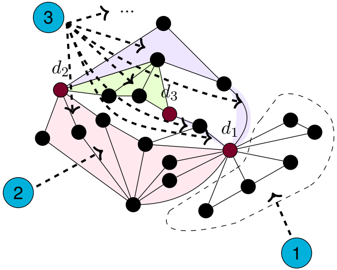

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Summary: Bounding Kernel Size

Let D be sds of size k. There exists a maximal D*-region decomposition* $\mathfrak{R}$ such that:

1. $\mathfrak{R}$ has only at most $3k - 6$ regions (Alber, Fellows Niedermeier [2]);
2. There are at most $97 \cdot k$ vertices outside of any region;
3. Each region $R \in \mathfrak{R}$ contains at most 87 vertices.

Hence: $|V| = \bigcup_{v \in D} N(v) = 87 \cdot (3k - 6) + 97 \cdot k < 358 \cdot k$

Main Theorem

All reduction rules can be applied in poly/time, hence:

The Main Theorem

The SEMITOTAL DOMINATING SET problem parameterized by solution size admits a linear kernel on planar graphs. There exists a polynomial-time algorithm that, given a planar graph (G, k), either correctly reports that (G, k) is a NO-instance or returns an equivalent instance (G', k) such that $|V(G')| \leq 358 \cdot k'$.

Conclusions

Results:

- Given an overview over the status
- SEMITOTAL DOMINATING SET is $W[1]$ for *chordal, split* and *bipartite* graphs
- exists linear kernel of size $358 \cdot k$ when parameterized by solution size

Future Work:

- Improve kernel size and do an empirical evaluation
- Resolve complexities for *Circle*, *chordal bipartite* and *undirected path graphs*

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? Any Questions ?

$\cdots$ *Thank you for your attention!* $\cdots$

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